Identification of source velocities with Inverse Patch Transfer Functions method

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The identification of source velocities remains an important problem in noise control. For this purpose, several methods were developed such as Near-field Acoustic Holography (NAH) or inverse Boundary Elements Method (iBEM). An alternative method, based on the double measurement of pressure and particle velocity fields surrounding the source is presented. This method has been developed in the SILENCE European project framework. In this method, called inverse Patch Transfer Functions Method (iPTF), measurement and identification surfaces are divided into elementary areas called patches. Then, source velocities are computed from acoustic field and inversion of impedances matrices obtained by FEM. Theoretically, this method presents two main advantages: it can be applied to sources with complex 3D geometries and measurements can be carried out in a non-anechoic environment even in the presence of other stationary sources. In the present paper, theoretical background of iPTF is exposed and results on a source with simple geometry (an L-shaped plate) are presented and discussed.

1 Introduction

Several holographic methods have been developed in acoustics to obtain source velocities in order to identify a source of noise when direct measurements are unavailable or predict the radiation of a complex structure. Among all these methods, one can find the Nearfield Acoustical Holography (NAH) or the inverse Boundary Elements Method (iBEM).

The NAH, developed by Maynard et al. [1], enables to determine the three-dimensional source velocity field by the measurement of the pressure field on a hologram near the vibrating source. However, the NAH is only applicable to simple geometries (planes, cylinders and spheres) and the boundary effects oblige to perform the measurements in the acoustical nearfield.

Contrary to the NAH, the iBEM is based on the numerical evaluation and inversion of transfer matrices. This last point can be problematic insofar as the transfer matrices are often ill-conditioned and thus require regularization. Furthermore, the iBEM can require a prohibitive amount of measurement if the reconstruction of acoustic quantities is performed on an arbitrary surface.

To overcome the limitations of NAH and iBEM, a hybrid method was developed [2]. Nevertheless, like other identification methods (NAH, iBEM...), this method requires a regularization of matrices because of the measuring incompleteness and uncertainty.

In this paper, the inverse Patch Transfer Functions Method (iPTF) is introduced to identify source velocities on complex surfaces. This inverse formulation is based on the PTF method [3], whereof purpose is to predict the acoustic pressure inside and outside a cavity containing sources and apertures by substructuration of the acoustical domain (interior and exterior). The subdomains are coupled by their common surfaces, divided into elementary areas called patches, where impedances matrices are defined.

The iPTF method can be applied when standard methods of measurements, such as laser vibrometer, can not be used due to the complexity of the geometry, thanks to the double measurements of pressure and particle velocity fields. Furthermore, the iPTF method use the Finite Element Method as a solver, since the modal basis of the interior domain is necessary to compute impedances matrices.

An experimental validation of the iPTF method is presented in the present method to prove the ability of the method to identify properly source velocities.

2 Theoretical background of the iPTF method

The iPTF method consists in solving Eq. (1) corresponding to the interior subdomain in the PTF formulation.

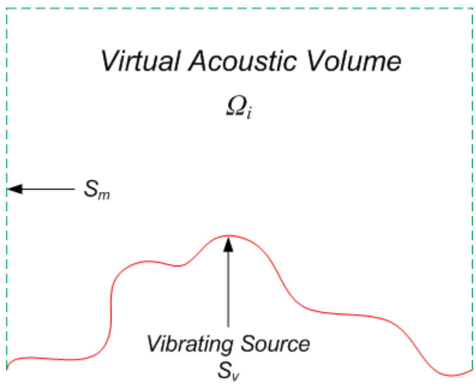

Fig.1 Basic vibro-acoustic problem.

$$\begin{cases} \Delta p(M) + k^2 p(M) = 0 \;\; \forall\, M \in \Omega_i \\ \dfrac{\partial p}{\partial n}(M) = -j\omega \bar{V}_n \;\; \forall\, M \in S_m \cup S_v \end{cases} \tag{1}$$

As expressed in Eq. (1), we have to solve the Helmholtz equation in a virtual acoustic volume Ω_i, while respecting the inhomogeneous Neumann boundary condition on the vibrating surface S_v (cf. Fig.1). The virtual surface S_m and the vibrating surface S_v are then divided into N and P patches respectively.

In the PTF method, the mean acoustic pressure on a patch j is obtained by the superposition of pressures due to the vibrating surface S_v and to the influence of the other patches of the virtual surface S_m (Eq. (2)).

$$\langle p_j \rangle = \sum_{k=1}^{P} Z_{jk} \langle v_k \rangle + \sum_{i=1}^{N} Z_{ji} \langle v_i \rangle \tag{2}$$

For a sake of simplicity, Eq. (2) is rewritten under matrix form (Eq. (3)).

$$\{P_j\} = [Z_{jk}]\{V_k\} + [Z_{ji}]\{V_i\} \tag{3}$$

$$Z_{ji} = \sum_n \frac{j\omega\rho c^2 s_i}{\Lambda_n\left(\omega_n^2 - \omega^2 + j\eta_n\omega_n\omega\right)} \langle\phi_n\rangle_i \langle\phi_n\rangle_j \qquad (4)$$

$$Z_{jk} = \sum_n \frac{j\omega\rho c^2 s_k}{\Lambda_n\left(\omega_n^2 - \omega^2 + j\eta_n\omega_n\omega\right)} \langle\phi_n\rangle_k \langle\phi_n\rangle_j \qquad (5)$$

Where $\{P_j\}$ is the mean acoustic pressure on the patch j, $[Z_{jk}]$ is the impedance matrix between a source patch k and a reception patch j of the coupling surface, $\{V_k\}$ is the mean source velocity, $[Z_{ji}]$ is the impedance matrix between an exited patch i and a reception patch j of the coupling surface and $\{V_i\}$ the mean coupling velocity. The impedance matrices $[Z_{jk}]$ and $[Z_{ji}]$ are obtained by modal expansion of the virtual acoustic cavity (Eq. (4) and Eq. (5)).

The iPTF formulation is then obtained by the inversion of Eq. (3) (Eq. (6)).

$$\{V_k\} = [Z_{jk}]^{-1}\left(\{P_j\} - [Z_{ji}]\{V_i\}\right) \qquad (6)$$

In Eq.(5), only the source velocities $\{V_k\}$ are unknown, since the impedances matrices $[Z_{jk}]$ and $[Z_{ji}]$ are numerically computed and the coupling pressures $\{P_j\}$ and the virtual velocities $\{V_i\}$ are measured with a Pressure-Velocity probe. One important point is that velocity on the virtual surface can be due to direct field from the vibrating surface as well as reflected sound by obstacles placed outside the virtual cavity. Thus, this method is not restricted to anechoic environment.

3 Application of the iPTF method on a simple geometry

3.1 Experimental setup

An L-shaped plate is used to validate the iPTF method. As presented in Fig.2, the setup is a steel box topped by a wooden formwork and placed on a concrete block to limit the amount of measurements by the creation of physical rigid wall conditions. A shaker excites the small surface of the box (cf. Fig.3) with a random white noise.

Fig.2 Experimental setup.

A virtual acoustic volume is then defined around the L-shaped plate as well as the measurement patch mesh (cf. Fig.3). Pressure and velocity data measured at the center of gravity of each patch represent the spaced averaged quantities on the patch surface. The aim of this experimentation is to compare laser vibrometer measurements, used as reference, with identified source velocities. Measurements are performed in a non-anechoic chamber and far from the acoustic source (130 mm).

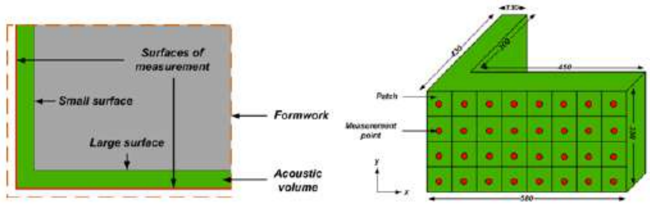

Fig.3 Definition of the virtual acoustic volume and patch mesh.

3.2 Identification of source velocities on the L-shaped geometry

iPTF method requires the inversion of impedance matrices $[Z_{jk}]$ (Eq.(6)), but this is problematic since $[Z_{jk}]$ are ill-conditioned rectangular matrices. Thus, a pseudo inversion has to be performed instead of a classic one. As a first step, iPTF identification was performed without regularization (cf. Fig.4).

Obviously, the ill-conditioning of impedance matrices leads to poor results. Four phenomena can explain an ill-conditioned matrix. Ill-conditioned impedance matrices are mainly due to the lack of pressure information on the source surface, which is essential to avoid under-determination of the problem. The second one is connected to the resonance frequencies, since at these particular values the determinant of impedance matrices is small which generates conditioning peaks. The second and the third phenomena are numerical. On one hand, there is a risk of redundant information in impedance matrices if patch mesh is very fine and on the other hand, the order of the cavity modes have to be greater than the number of patches in a direction as shown by Ouisse et al in [3].

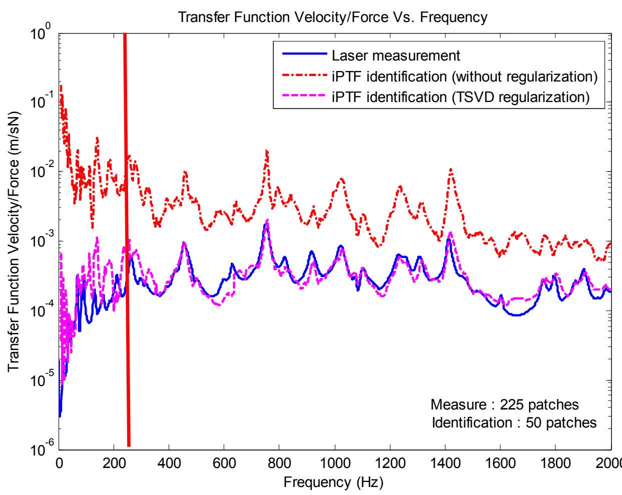

Fig.4 Identification of source velocity: (-) Reference (laser vibrometer), (-.-) iPTF identification without regularization, (- -) iPTF identification with TSVD identification for 225 measurement patches and 50 identification patches

As shown in Fig.4, a classical pseudo inversion is ineffective to obtain proper results. Regularization methods are commonly used to overcome ill-conditioned problems. In this study, Truncated Singular Values Decomposition (TSVD) regularization is used [4].

The TSVD regularization is based on the L-curve to find the optimal regularization parameter (cf. Fig.5). The principle is to plot at each frequency the norm of the regularized solution η versus the norm of the corresponding residue ρ in a log-log scale in order to find the optimal parameter. This parameter is given by the corner of the L-curve. However, the corner of the L-curve is hard to determine.

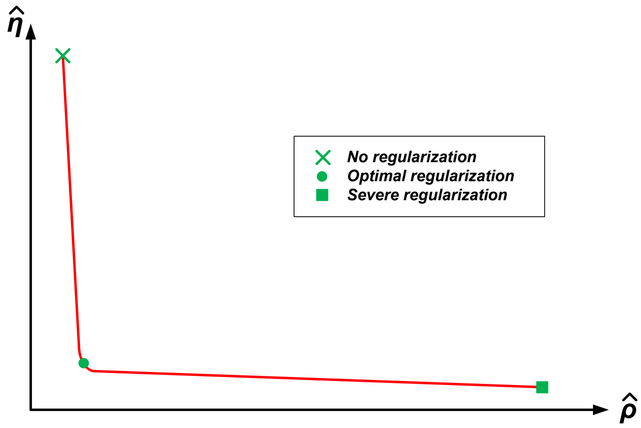

Fig.5 L-curve

To determine the optimal number of singular values, we used an indicator defined by Leclère in [5]. This indicator is constructed from the normalized norm of the solution η_n and the normalized norm of the residue ρ_n (Eq.(7) and Eq.(8)).

$$\rho_n(\beta) = \frac{\left\|\{p_j\} - [Z_{ji}]\{v_i\} - [Z_{jk}]\{v_k\}\right\|_2}{\left\|\{p_j\} - [Z_{ji}]\{v_i\}\right\|_2} \qquad (7)$$

$$\eta_n(\beta) = \frac{\|V_k\|_2}{\|V_k\|_2^0}\left(1 - \rho_n(0)\right) \qquad (8)$$

Where $\|V_k\|_2^0$ is the norm of non-regularized solution, $\rho_n(0)$ is the norm of non-regularized normalized residue and β the regularization parameter.

The indicator is thus given by Eq. (9):

$$I = \eta_n(\beta) + \rho_n(\beta) \qquad (9)$$

The corner of the L-curve corresponds to the minimum of the indicator.

This method is used to regularize the impedance matrices $[Z_{jk}]$ and the result of iPTF identification is presented in Fig.4.

The regularized results are consistent with the laser measurements except in very low frequency. Actually, below 100 Hz, Pressure-Velocity measurements are drowned in the background noise. Poor results are notably observed below the first virtual cavity mode at 206 Hz. Actually in iPTF method, evanescent waves are modeled by cavity modes. Consequently, below the first cavity mode, evanescent waves are not taken into account in the formulation. From 206 Hz, the identification error is more or less 3 dB, which is satisfying, especially that measurements have been made in a non-anechoic chamber (presence of the reflected sound from the room).

Moreover, the iPTF method enables to establish maps of source velocity field on the source surface, as shown in Fig.6 and Fig.7.

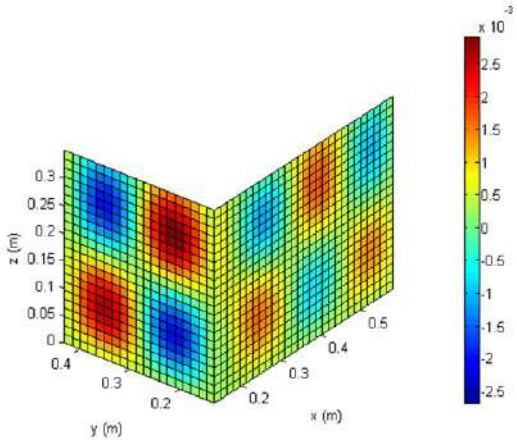

Fig.6 Source velocity field measured with a laser vibrometer at 1022Hz

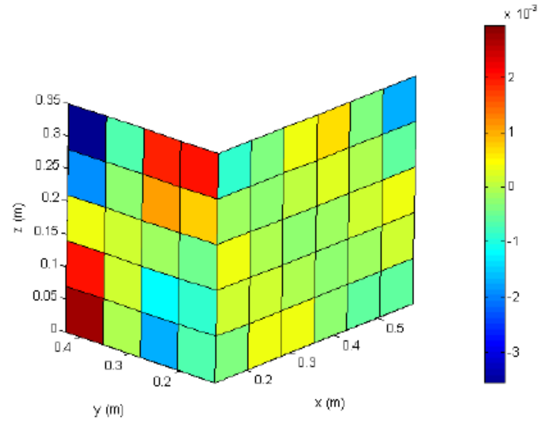

Fig.7 Source velocity field identified with the iPTF method for 50 identification patches at 1022Hz

Fig.6 and Fig.7 show that iPTF method gives consistent results even if the patch mesh is coarse. However, a better resolution is preferable to observe the source velocity field. For this purpose, a finer identification patch mesh has to be defined. Fig.8 and Fig.9 present an example of iPTF identification maps computed at 1756 Hz.

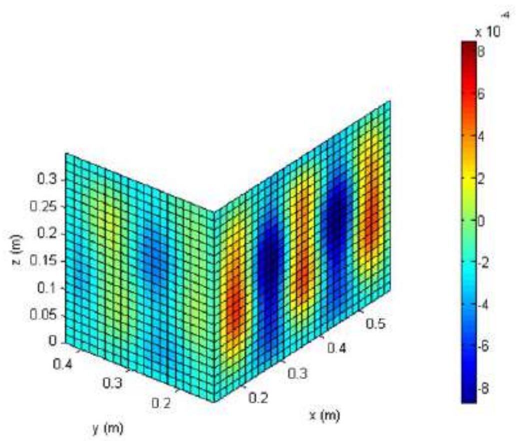

Fig.8 Source velocity field measured with a laser vibrometer at 1756Hz

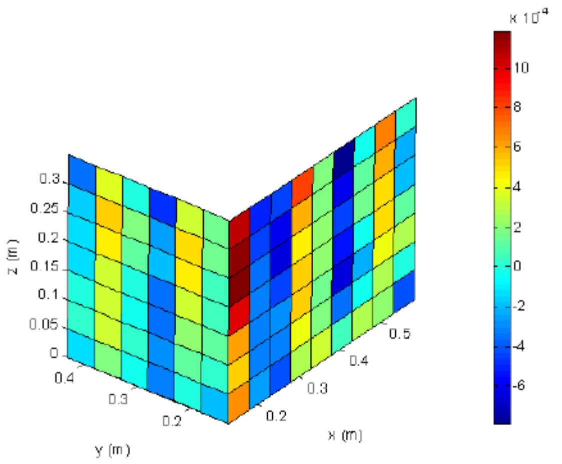

Fig.9 Source velocity field identified with the iPTF method for 105 identification patches at 1756Hz

In this case, the identification maps agree still well with laser vibrometer measurements.

All these results show that the iPTF method enables to identified and reconstruct properly the source velocity field.

4 Identification of source velocities in presence of a stationary source

The aim of this section is to prove the ability of the method to identify source velocities in presence of a stationary polluting source. For this purpose, source velocity field is identified on the large surface of the L-shaped plate (cf. Fig.10).

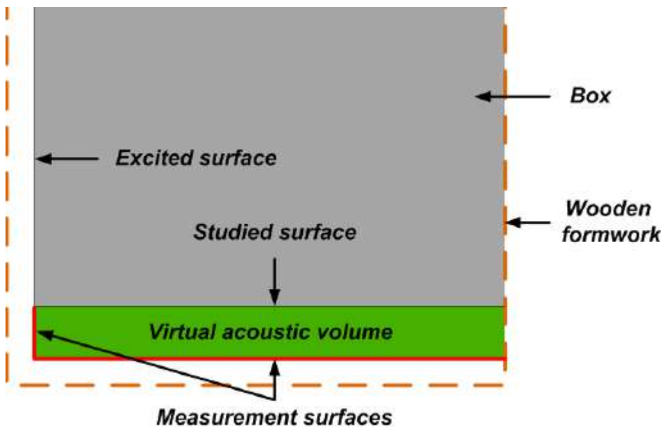

Fig.10 Definition of the virtual acoustic volume

The excited surface is acted as a polluting stationary source, because located outside of the virtual acoustic cavity. iPTF results are given in Fig.11 and Fig.12.

Fig.11 shows a good agreement between laser measurement and iPTF identification except below 378 Hz, which is the first virtual cavity mode, as explained in the previous section. Furthermore, the comparison between the iPTF identification on the complete L-shaped plate and the large surface alone is satisfactory. Of course, a finer patch mesh is preferable to obtain more accurate maps.

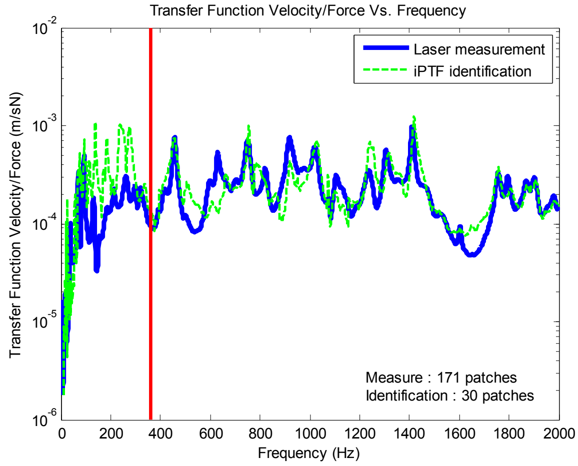

Fig.11 Identification of source velocity: (-) Reference (laser vibrometer), (- -) iPTF identification with TSVD regularization for 171 measurement patches and 30 identification patches

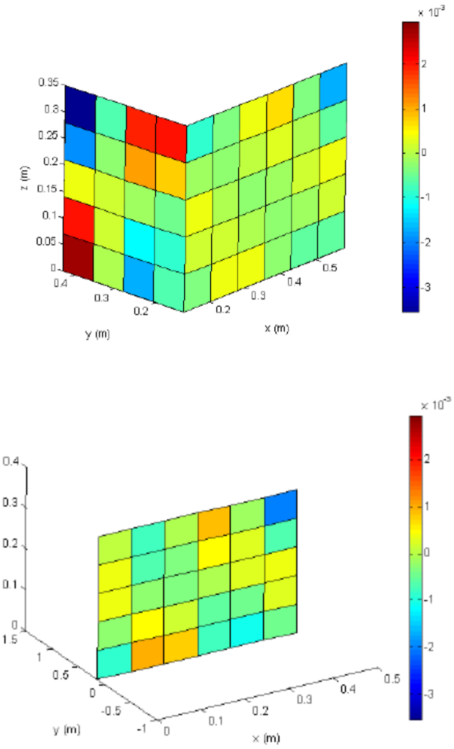

Fig.12 Distribution of source velocity – Comparison between iPTF identification on the L-shaped geometry and the large surface alone at 1022 Hz

5 Conclusion

The iPTF method enables to identify source velocity field on a complex tridimensional structure. Through a simple geometry, we show the ability of method to identify and reconstruct the source velocity field. In other words, the iPTF results are consistent both in amplitude and distribution.

The second advantage of the method is to be independent of the environment, if external sources are stationary. Finally, the double measurement of pressure and velocity fields can be made far from the source while identifying properly source velocity.

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