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INTRODUCTION OF RESIDUAL MODES CONCEPT IN THE PATCH TRANSFER FUNCTIONS METHOD TO MODEL THE STRUCTURE-ACOUSTIC COUPLING IN HEAVY FLUID

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The Patch Transfer Functions (PTF) method is used in this paper to model the structure-acoustic coupling in heavy fluid. This substructuring method allows determining the response of a coupled system with a reasonable computational cost. Finite Element Method is used in the proposed approach to compute the PTF of uncoupled subsystems, which are finally coupled through their common surface divided into elementary areas called patches. The basic equations of PTF method are reminded before introducing the residual modes concept in the cavity-PTF computation. Contrary to residual modes technique classically used in FEM, residual cavity modes defined in the proposed approach are independent of the structural system and allows modifications of structural modes without recalculate residual cavity modes. They allows accelerating the convergence of the method, especially in case of strong coupling. A comparison with standard FEM is proposed for a plate coupled to a water-filled cavity and excited by a point force. The convergence of FEM method is studied as well as that of the PTF method with and without residual modes. Finally, the combination of the PTF method with the residual mode concept gives proper results for a coarser interface mesh than that used in standard FEM.

1. Introduction

The vibro-acoustic coupling is often met in industrial applications and consequently remains an important issue. This interaction has been extensively studied through analytical modellings providing comprehensive formulations [1, 2, 3]. Although providing physical insights, these models are limited to simple system geometries.

As soon as industrial problems are considered the Finite Element Method (FEM) replaces analytical methods. However, a complex geometry leads to large FE models which are time consuming. In such conditions, dynamic reduction methods like Component Mode Synthesis [4] are used. These methods based on the modal approach allows reducing the size of the FE model by describing the dynamics of the coupled system with a reduced but sufficient number of degrees of freedom. Unfortunately, in case of strong coupling the use of *in vacuo* modal bases leads to poor convergence properties if the coupling between high order modes of a subsystem with low order modes of the other one is miscalculates. For this purpose, pseudostatic correction [5, 6] or residual modes concepts [7, 8] have been introduced to improve the convergence of FEM.
In the present study, the forced response of simply supported plate backed by a water-filled cavity is computed by the Patch Transfer Functions (PTF) method [9], which is based on substructuring and impedance and mobility approaches [10]. The residual modes concept is introduced in the method to improve its convergence and keep advantages of substructuring. In this paper, the theoretical background of the method is first briefly reminded. The convergence of FEM method is then studied as well as that of the PTF method with and without residual modes. Obtained results confirm expected advantages of the proposed method, namely proper predictions of structure-acoustic response with a coarser interface mesh than that used in standard FEM.

2. Vibro-acoustic problem

A structure backed by a water-filled cavity with rigid wall and excited by a point force F_s on its surface S_v is considered Fig. 1.

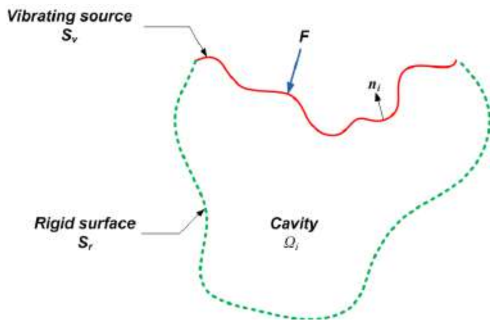

Figure 1. Basic vibro-acoustic problem

The structure is governed by its equation of motion with boundary conditions imposed on its boundary ∂S_v. Furthermore, this structure is subject to a point force F_s and the pressure-induced force F_c describing the action of the cavity on the structure. Finally, we suppose that the structure displacement $u_s(M,t)$ is harmonic and satisfies Eq. 1.

$$\begin{cases} L_s.u_s(M,\omega) = F_s + F_c & \forall\, M \in S_v \\ \text{Boundary conditions on } \partial S_v \end{cases} \tag{1}$$

The acoustic cavity is governed by the Helmholtz equation with rigid walls boundary conditions imposed on the rigid surface S_r. Moreover, a normal harmonic displacement $u_n(M,t)$ resulting from the vibration of the structure is imposed on the vibrating surface S_v. Finally, the acoustic pressure in the cavity $p(M,t)$ is supposed harmonic and satisfies Eq. 2.

$$\begin{cases} \Delta p\,(M) + k^2 p\,(M) = 0 & \forall\, M \in \Omega_i \\ \dfrac{\partial p}{\partial n}\,(M) = 0 & \forall\, M \in S_r \\ \dfrac{\partial p}{\partial n}\,(M) = \rho\omega^2 u_n(M) & \forall\, M \in S_v \end{cases} \tag{2}$$

The weak form of Eqs. 1 and 2 leads to the FE model gives by Eq. 3.

$$\left(\begin{bmatrix} K_s & C \\ 0 & K_f \end{bmatrix} - \omega^2 \begin{bmatrix} M_s & 0 \\ -C^T & M_f \end{bmatrix} \right) \begin{Bmatrix} U_s \\ P \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \tag{3}$$

In Eq. 3, the subscript s corresponds to the structure, while the subscript f corresponds to the fluid domain, i.e. the acoustic cavity. Furthermore, the structure-cavity coupling is introduced through the coupling matrix C.

3. Patch Transfer Functions method

3.1 Definition of the basic quantities

As exposed in [9, 11, 12], the PTF method is based on substructuring and impedance and mobility concepts to compute with a reasonable computational cost the response of coupled systems. One of the main adantages of this method is to solve each subsystem indenpendently of the others. By this way, a modification of one of the subsystems only leads to update this subsystem, provided the geometry of the structure-cavity interface remains unchanged.
In the case presented in this paper, the structure-cavity system is divided into two subsystems, the structure and the acoustic cavity. Then, the interface surface S_v between the subsystems is divided into elementary areas called patches, where the PTF are defined for each subsystem.

For the structure, the PTF is the mobility Y_{jk} between an excited patch k and a receiving patch j defined as the ratio of the mean structural velocity $\bar{v}_j^s$ on the patch j and the mean pressure $\bar{p}_k^s$ on the patch k:

$$Y_{jk} = \frac{\bar{v}_j^s}{\bar{p}_k^s} \tag{4}$$

For the acoustic cavity, the PTF is the impedance Z_{jk} defined as the ratio of the mean acoustic pressure $\bar{p}_j^c$ on the patch j and the mean normal velocity $\bar{v}_k^c$ on the patch k:

$$Z_{jk} = \frac{\bar{p}_j^c}{\bar{v}_k^c} \tag{5}$$

3.2 Structure-cavity coupling

To couple the subsystems, continuity equations are written for each patch j belonging to the interface surface S_v as expressed in Eq. 6.

$$\begin{cases} \bar{p}_j^s = \bar{p}_j^c = \bar{p}_j \\ \bar{v}_j^s = \bar{v}_j^c = \bar{v}_j \end{cases} \tag{6}$$

Then, the linearity of the problem is used to compute the mean patch normal velocity $\bar{v}_j^s$ and the mean patch pressure $\bar{p}_j^c$. The mean patch normal velocity $\bar{v}_j^s$ is defined as the superposition of the *in vacuo* structural velocity $\tilde{\bar{v}}_j^s$ and the normal velocity $Y_{jk}^s \bar{p}_k$ due to the pressure induced by the acoustic cavity on the structure, while the mean patch pressure $\bar{p}_j^c$ is defined as the superposition of the pressure created by the acoustic source inside the cavity $\widetilde{\bar{p}}_j^c$ and the pressure due to structural vibrations $Z_{jk}^c \bar{v}_k$ (see Eq. 7).

$$\begin{cases} \bar{v}_j^s = \widetilde{\bar{v}}_j^s + Y_{jk}^s \bar{p}_k \\ \bar{p}_j^c = \widetilde{\bar{p}}_j^c + Z_{jk}^c \bar{v}_k \end{cases} \tag{7}$$

In absence of acoustic sources in the cavity, the introduction of Eq. 6 in Eq. 7 allows determining the coupling normal velocity $\bar{v}_j$ as expressed in Eq. 8.

$$\bar{v}_j = \left(\mathbb{I} - Y_{ji}^s Z_{ik}\right)^{-1} \widetilde{\bar{v}}_k^s \tag{8}$$

Where $\mathbb{I}$ is the identity matrix.

3.3 PTF computation

From a practical point of view, the PTF are derived from the FE formulation expanded on the modal basis of each subsystems.

For the structure-PTF, we have to compute the normal structural velocity when the structure is excited by a constant pressure $\bar{p}_k$ on the patch surface S_k as defined by Eq. 4. Consequently, for each patch k of the structure, the FE model given by Eq. 9 has to be solved.

$$\left[K_s - \omega^2 M_s\right]\{U_s\} = \{F_k\} \tag{9}$$

Where the excitation term F_k corresponding to the constant pressure $\bar{p}_k$ imposed on the patch k is given by Eq. (10).

$$F_k = \begin{cases} \frac{A_k}{N}\bar{p}_k & \text{on } S_k \\ 0 & \text{elsewhere} \end{cases} \tag{10}$$

Where A_k is the area of the patch k, N the number of FE nodes included in the patch k.

To solve Eq. 10, the standard mode expansion is used, which means that the displacement $\{U_s\}$ is expanded on its *in vacuo* structural modes as in Eq. 11.

$$\{U_s\} = [\psi_n]\{u_n\} \tag{11}$$

Where $[\psi_n]$ is the modal shapes matrix and $\{u_n\}$ is the modal amplitudes vector.

By this way, the reduced system given by Eq. 12 is obtained and solved at each frequency by standard procedure to finally determine the structural displacement $\{U_s\}$.

$$\left[K_{sn} - \omega^2 M_{sn}\right]\{u_n\} = \{F_n\} \tag{12}$$

When repeating this procedure for each patch k, the structural velocity matrix $[V]$ is derived and the structure-PTF Y_{jk} is finally obtained by averaging the normal velocities for nodes included in the receiving patch j.

The cavity-PTF are similarly derived from the FE model of the acoustic cavity as expressed by Eq. 13.

$$\left[K_f - \omega^2 M_f\right]\{P\} = \{Q_k\} \tag{13}$$

Where the excitation term Q_k corresponding to the constant normal velocity $\bar{v}_k^c$ imposed on the patch k is given by Eq. 14.

$$Q_k = \begin{cases} -j\omega\frac{A_k}{N}\bar{v}_k^c & \text{on } S_k \\ 0 & \text{elsewhere} \end{cases} \tag{14}$$

It is important to notice that in above equations, the PTF have been defined without introducing residual modes. In the following section, it will be shown that the convergence of the PTF method is difficult to achieve for acoustic cavities filled with a heavy fluid like water. In this situation, the residual modes are computed to enrich the modal basis of the cavity and consequently improve the convergence of the proposed method.

4. Simulation of the structure-cavity coupling in heavy fluid

In this section, the PTF method is used to compute the response of structure-cavity system and compared with standard FEM calculation. For this purpose, we consider in this section a parallelepiped water-filled cavity with rigid walls of dimension $L \times b \times H = 0.5 \times 0.4 \times 1\ m^3$ coupled to a rectangular simply supported plate excited by a point force F in the frequency range of interest $[0; 100]$ Hz as presented in Fig. 2. The characteristics of the plate are: $L \times b = 0.5 \times 0.4\ m^2$, thickness $h = 0.002\ m$, $E = 2.1 \times 10^{11}\ Pa$, $\nu = 0.3$ and structural damping factor $\eta = 0.02$.

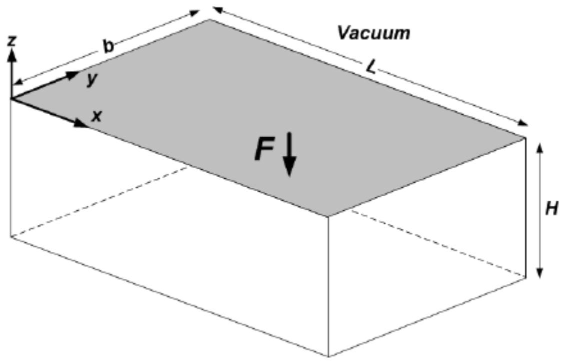

Figure 2. Definition of the rectangular simply-supported plate excited by a point force F and coupled to a parallelepiped water-filled cavity

4.1 Influence of the discretization in FEM

The interface mesh size is a key parameter to properly describe the response of a coupled system. To study the influence of this parameter, a direct resolution of the structure-cavity system is performed for four interface mesh criteria using NASTRAN. These criteria are based on the smallest wavelength λ of uncoupled plate and cavity and computed at the maximal studied frequency. For each mesh criterion ($\frac{\lambda}{6}$, $\frac{\lambda}{12}$, $\frac{\lambda}{28}$ and $\frac{\lambda}{60}$) the mean square velocity of the plate coupled to the water-filled cavity is computed from Eq. 8 as presented in Fig. 3. This figure shows that a very fine interface mesh is required since the convergence of standard FEM occurs for a $\frac{\lambda}{28}$ mesh criterion.

This poor convergence in heavy fluid is explained by the decrease in flexural wavelength due to the added mass effect of the acoustic cavity on the plate.

4.2 Influence of residual modes in the PTF method

In light fluid, Ouisse *et al.* [11] have proven that a $\frac{\lambda}{2}$ patch mesh criterion for the coupling surface was sufficient to obtain proper results. However, in heavy fluid, the decrease of the flexural wavelength of the plate coupled to an acoustic cavity has to be taken into account. Consequently, we used a $\frac{\lambda}{6}$ patch mesh criterion, which corresponds in the present case to 56 patches.

In the example presented in Fig. 4, 5 structural modes, corresponding to modes computed up to 205 Hz, have been used to compute the structure-PTF Y^s_{jk}, which is sufficient to ensure its convergence. A first computation using 2 cavity modes, corresponding to modes computed up to 750 Hz,

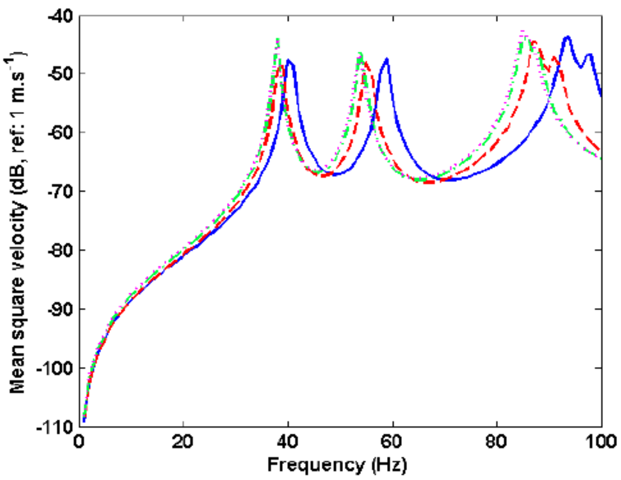

Figure 3. Convergence of FEM in heavy fluid - Comparison of mean square velocity obtained by direct resolution for 4 mesh criteria: (-) $\frac{\lambda}{6}$, (- -) $\frac{\lambda}{12}$, (-.-) $\frac{\lambda}{28}$ and (. . .) $\frac{\lambda}{60}$

was performed with the PTF method. Results presented in Fig. 4 shows a poor convergence of the method, which consequently requires a great number of cavity modes to converge. This poor convergence also indicates that the contribution of the high-order cavity modes is very important when considering a strong coupling. This limitation can be alleviated by using residual modes. Actually, residual modes do not correspond to physical modes but allows enriching the modal basis by a set of functions describing the high-order modes contribution.

To keep the substructuring aspect of the PTF method, the residual modes defined here corresponds to the quasi-static response of the cavity excited by a constant normal displacement imposed on a patch p as expressed in Eq. 15. By this way, the number of residual modes is equal to the number of patches. One can also notice that this definition of the residual modes differs from that classically used when solving structure-acoustic problems, since the residual modes for an acoustic cavity generally corresponds to the quasi-static response of the cavity excited by structural normal modes.

$$\left[K_f - \omega_c^2 M_f\right] \{P_r\} = \{Q_p\} \tag{15}$$

Where the excitation term Q_p corresponding to the unit normal displacement $\bar{u}_k^c$ imposed on the patch k is given by Eq. 16.

$$Q_p = \begin{cases} \omega_c^2 \frac{A_p}{N} \bar{u}_p^c & \text{on } S_p \\ 0 & \text{elsewhere} \end{cases} \tag{16}$$

The value of ω_c in Eqs. 15 and 16 used in this example corresponds to 20% of the first non-zero cavity mode ($\approx$ 150 Hz). In fact, this value can be arbitrarily chosen as long as the value of this parameter remains below the frequency of the last physical modes retained to compute the cavity-PTF.

The new reduction basis containing the N retained physical modes in the original basis enriched by the N_r residual modes has to be reorthogonalized to keep valid the mass and stiffness orthogonality relation (see [13]).

In the example presented in this paper, the new reduction basis contains contains 2 physical modes and 56 residual modes. When using this new reduction basis, the mean square velocity obtained by the PTF method for a $\frac{\lambda}{6}$ patch mesh criterion is in a very good agreement with the reference result computed by FEM. This shows that the combined use of PTF method and residual modes concept allows an effective time-saving computation compared to standard FEM.

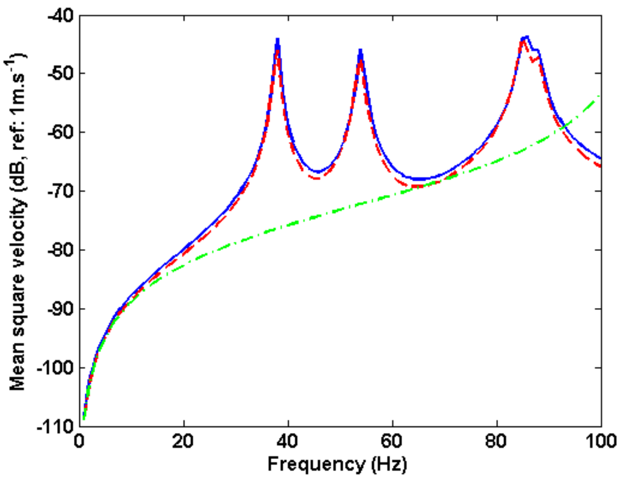

Figure 4. Residual shapes effect - Comparison of mean square velocity for: (-) FEM solution ($\frac{\lambda}{28}$), (-.-) PTF method without residual shapes ($\frac{\lambda}{6}$) and (- -) PTF method introducing residual shapes ($\frac{\lambda}{6}$)

5. Conclusion

The PTF method has been used to solve the structure-acoustic coupling in heavy fluid. The introduction of the residual modes technique in this approach allows keeping the advantages of the PTF method while improving its efficiency. Indeed, the PTF method allows computing the response of a coupled system with a reasonable computational cost by using substructuring and impedance and mobility concepts. The present work shows that the combined used of both techniques gives very satisfaying results with a coarser interface mesh than that used in the FEM. Finally, although the method is presented on a simple case, it can be directly extended to more complex coupled systems, since the formulation is based on the FE model of each subsystem.

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