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Modelling of sound transmission through ship structures using the Patch Transfer Functions approach

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ABSTRACT

Finite Element model can be used to predict the vibro-acoustic behaviour of elastic structures coupled by water filled cavities. These calculations can be time consuming due to the large size of the problem. To overcome this drawback, one proposes to use the Patch Transfer Functions (PTF) approach to partition the global problem in different sub-problems. Then, one studies different manners of partitioning the problem and different approaches to estimate the PTF of each sub-problem. One will show that the partitioning outside the near field of the structures permits to reduce the number of patches for frequencies below the critical frequency. On another hand, a non standard modal expansion based on a symmetrical formulation of the fluid-structure problem permit to calculate the PTF with enough accuracy to ensure the convergence the PTF method and to save computing time compared to direct resolution. This approach is an efficient tool for modelling of sound transmission through ship structures in the mid-frequency for instance.
Keywords: fluid-structure interaction, heavy fluid, numerical methods

1. INTRODUCTION

The modeling of the interaction between an elastic structure and water filled cavities is of interest in many applications, especially in the nuclear and the naval industries. The finite element methods and boundary element analyses are relevant in an industrial context to model fluid-structure interaction since they allow considering the structures and interfaces of arbitrary shapes. However, these models lead to non-symmetric matrices and the number of degrees of freedom to be taken into account (ie the number of unknowns) increases rapidly with frequency. The direct solution of the corresponding linear system may then be too time-consuming. Alternative methods [1-9] have been proposed to overcome this obstacle. Some [1,2] use the modes of the uncoupled subsystems (i.e. in-vacuo modes of the structure and modes of the cavity with rigid walls) and the addition of residual modes to improve the convergence of modal expansions. Others [3-7] consist to symmetrize the matrices describing the fluid-structure interaction in order to extract easily the coupled modes in a second step.

In this paper, we propose to sub-structure the fluid-structure problem from the PTF approach (Patch Transfer Functions) [10-12] for a frequency range well below the critical frequency of the structure, f_c. This PTF approach is based on substructuring through surfaces divided in elementary areas called patches and it consists in studying each subsystem independently in order to build a set of transfer functions defined by using mean values on the patches, called Patch Transfer Functions. Then, assembling PTF by using the superposition principle for linear passive system and using the continuity relations lead to a fast resolution of the coupled problem. This method was successfully applied to address problems of acoustic radiation for the automotive industry. We wish extending it here to the

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interaction between a structure and a heavy fluid cavity. The principle of the method is recalled in section 2. The PTF approach making no assumptions about the strength of the subsystems coupling, we will discuss in section 3 the possibility of sub-structuring the problem in the near field of the structure or outside this zone. A basic analytical model allows defining a criterion that estimates the optimal position of the coupling surface surface in order to the pressure distribution varies spatially as the acoustical wavelength on the coupling surface. It permits to define the patch size criterion based on the acoustic wavelength that reduces the number of patches comparing to a substructuring in the near field of the plate. These investigations are validated on a basic test case. This approach implies the calculation of PTFs for a subsystem made up of a mechanical structure and a surrounding fluid. The computation of these PTFs is not easy, since the FE matrix associated to this subsystem is non-symmetric. To overcome this difficulty, a non standard modal expansion involving a symmetrization of the finite element equations of the fluid-structure problem is proposed in section 4.

2. PTF approach

Let us consider the vibro-acoustic problem as illustrated Fig. 1 on an academic case. An elastic structure is excited by a harmonic point force and coupled to a rigid-wall cavity filled with a fluid at rest. The global system is decomposed into two subsystems by cutting the volume through a coupling surface, S_c. This surface is then divided into N patches. The position of this coupling surface and the number of patches will be discussed in section 3.

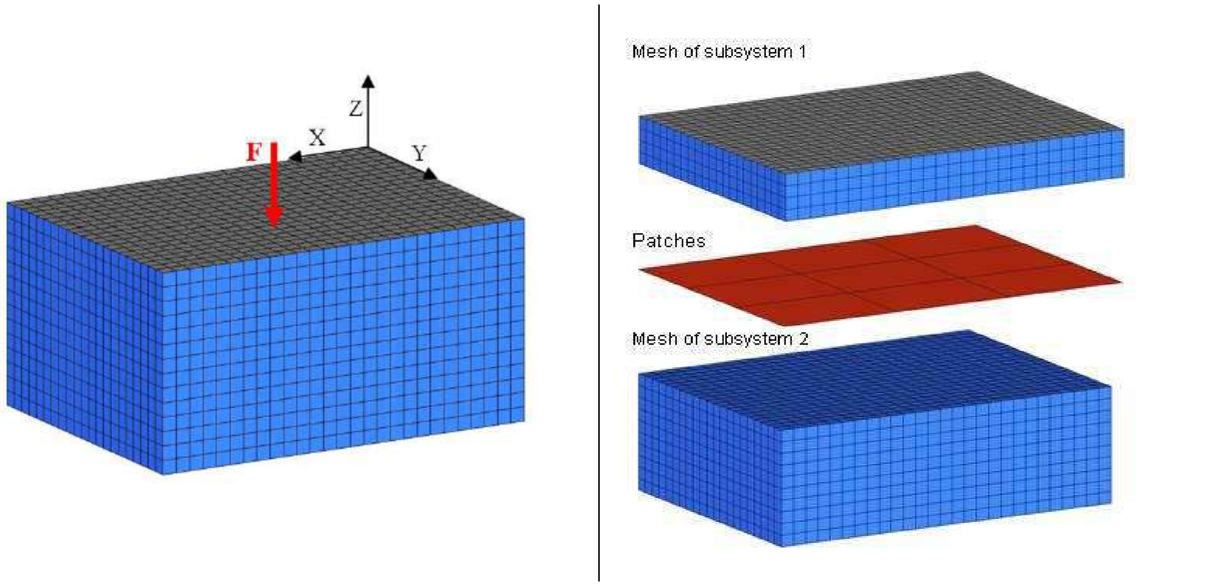

Figure 1. Structure-Cavity problem and PTF substructuring.

To define the Patch Transfer Functions, each subsystem is considered independently. For the subsystem α (α=[1,2]), a constant normal velocity $\bar{v}_i^\alpha$ is prescribed on the patch i of surface ∂S_i, whereas a null normal velocity is prescribed on the other patches. One defines the PTF of subsystem α by:

- the Patch Transfer Functions between the patch i and the patch j, Z_{ij}^α :

$$Z_{ij}^\alpha = \frac{\bar{p}_j^\alpha}{\bar{v}_i^\alpha \partial S_i}, \tag{1}$$

where $\bar{p}_j^\alpha$ is the space-averaged pressure on the patch j.

- the Patch Transfer Functions between the patch i and the point M inside the subcavity, Z_{iM}^α :

$$Z_{iM}^\alpha = \frac{p_M^\alpha}{\bar{v}_i^\alpha \partial S_i}, \tag{2}$$

where p_M^α is the resulting pressure at point M.

For the subsystem 1 which is excited by the external source, one defines also the blocked pressure of patch *i*, $\tilde{p}_i^1$ as the mean of the resulting pressure on patch *i* due to the external force *F*.

By using the superposition principle for linear passive and writing the continuity conditions on the *N* patches, we obtains (see [10]) a system of *N* linear equations having the *N* patch velocities as unknowns:

$$\sum_{j=1}^{N}\left[\left(Z_{ji}^{1}+Z_{ji}^{2}\right)\partial S_{j}v_{j}^{2}\right]=\tilde{p}_{i}^{1},\quad \forall i\in\left[1,\dots,N\right]. \tag{3}$$

After solving this system, we can deduce (with the superposition principle for linear passive system), the pressure at point *M* of subsystem 1:

$$p_M^1=\tilde{p}_M^1-\sum_{j=1}^{N}Z_{iM}^{1}\partial S_i v_i^2\,, \tag{4}$$

and, the pressure at point *M’* of subsystem 2:

$$p_{M'}^2=\sum_{i=1}^{N}Z_{iM'}^{2}\partial S_i v_i^2\,. \tag{5}$$

The normal displacement at point *M”* of the mechanical structure (subsystem 1) may be deduced with the same process and using appropriate PTFs between the patches and the receiving point *M”*.

The PTF approach allows us to calculate the response of the global system from the knowledge of the PTFs of each uncoupled subsystem and inverting a square matrix whose dimensions correspond to the number of patches. PTFs can be calculated by different techniques depending on the considered subsystem (analytical, finite element, boundary element, Rayleigh integral, etc.). These calculations are performed on each subsystem, separately. Then, the computer resources for achieving these calculations are generally lower than those require for solving the global problem.

3. Position of the coupling surface

In this section, the position of the coupling surface S_c is studied. This surface defines the interface between two subsystems. Since no assumptions are made in the PTF formulation on the subsystems coupling, it is theoretically possible to partition the global system by any fictive surface.

Furthermore, a parametric study has shown that the patches should have a size lower than the half acoustic wavelength at the greater frequency of interest (i.e. patch size criterion λ/2) [10].

In the present paper, the structure is loaded by a heavy fluid. In general, in heavy fluid applications, the wavelength of the flexural motion of the structure is smaller than the acoustic wavelength, insofar as the critical frequency is very high. These two wavelengths will play a role in the fluid medium, depending of the distance from the structure. Indeed, in the near-field of the structure, the acoustic pressure varies according to the bending wavelength λ_f of the structure. Thus, the patch mesh criterion has to be based on the structural wavelength. However, it leads to an important number of patches. On the contrary, in the far-field of the structure, the acoustic pressure varies according to the acoustic wavelength λ_a. In this situation, a patch mesh criterion based on the half acoustic wavelength can be applied that limits the number of patches for frequencies below the critical frequency.

In order to define the portion of the fluid domain where one can assume that the acoustic pressure varies according to the acoustic wavelength, one studies the decay of evanescent waves in the fluid medium generated by an infinite flat plate equivalent to the considered structure. Based on a criterion of an attenuation of 10 dB of the evanescent waves, one obtains a minimal distance defined by:

$$Z_{\lim}=\frac{\ln(10)}{2\sqrt{k_f^{\,2}-k_0^{\,2}}}\,, \tag{6}$$

where k_f, is the wavenumber of bending of the structure and k_0, the acoustic wavenumber at the considered frequency.

To illustrate this discussion, one considers the test case shown on Fig. 1. It consists in a rectangular simply-supported plate excited by a point force *F* and coupled to a parallelepiped water filled cavity (stell plate: 2m x 1.5m; thickness, 17mm ; material : water filled cavity: 2m x 1.5mx1m). One proposes on Fig. 2 the patch size criteria $\lambda_f/2$, $\lambda_a/2$ and the limit distance Z_{lim} in function of the frequency for this test case. One can observe that if one considers a patch size of about 0.7 m, the criterion $\lambda_a/2$ is well respected up to 750 Hz, whereas it is only satisfy below 60 Hz for the $\lambda_f/2$ criterion.

Two partitions of the cavity are considered for the PTF calculations:

- The first where the coupling surface is positioned at 0.3 m from the plate, i.e. at a distance of the plate greater than Z_{lim} for frequencies above 50 Hz (see substructuring of Fig. 1);
- The second where the coupling surface is at 0.05 m from the plate, i.e. at a distance less than Z_{lim} , whatever the frequency in the frequency range [1 Hz - 750 Hz].

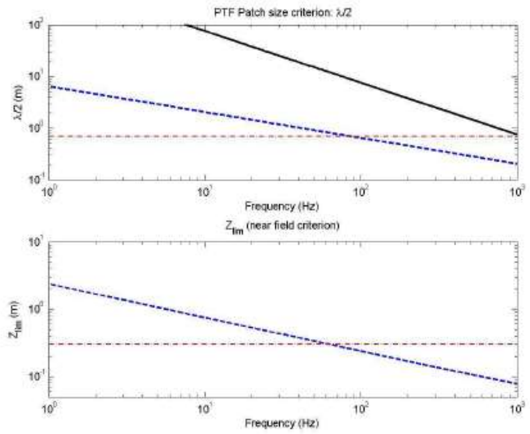

Figure 2. Patch size criteria λ/2 (upper) for the fluid medium (black) and for the plate structure (blue). Z_{lim} parameter defined by Eq. (10) (lower).

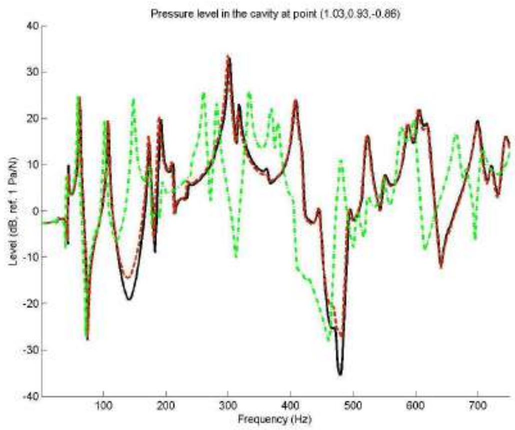

Figure 3. Comparison of the pressure level inside the cavity for three calculations: red line, PTF results with the first substructuring; green line, PTF results with the second substructuring; black line, direct FEM results (reference).

PTF calculations are achieved for these two sub-structurings (9 patches on the surface coupling). The PTFs of each subsystem are obtained by solving directly the equations related to the finite element model of each subsystem (SOL108 in NASTRAN code). A reference result for this test case is obtained by a direct resolution of the FE problem of the global system. A comparison of the two PTF results with the reference one is proposed on Fig. 3.

This comparison shows that the PTF calculation with the first substructuring gives results very close to the reference calculation over the entire frequency range while the PTF calculation with the second substructuring did not converge. The same results are obtained for others points inside the fluid domain and for points on the plate. It may be emphasized that the coupling surface of the PTF calculation with the first substructuring is located at a distance less than Z_{lim} for frequencies below 50 Hz, but the calculations converge at these frequencies because the size of the patches is less than $\lambda_f/2$.

In conclusion, this result shows that a patch mesh criterion based on the half acoustic wavelength can be considered while the coupling surface is located at an "optimal" distance defined from Eq. (6). In this section, the PTFs of each subsystem were obtained by a direct resolution of the FEM equations, which were time consuming. We propose in the next section a non-standard modal approach for solving the fluid-structure problem related to the subsystem 1 (structure + surrounding cavity).

4. A modal expansion method for a fluid-structure problem

Let us considered the finite element model (FE) of subsystem 1 composed by the plate and the surrounding fluid. The formulation (***U***, ***P***) model is written by (see [3]):

$$\begin{bmatrix} K_S & -A \\ 0 & K_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} - \omega^2 \begin{bmatrix} M_S & 0 \\ A^T & M_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix} \tag{7}$$

where: - U and P represent the nodal displacements and the nodal pressure;
- F are the nodal forces and Q the nodal volume velocities;
- M_S and K_S are the mass and stiffness matrices of the structure;
- M_F and K_F are the mass and stiffness matrices of the cavity;
- A are the fluid-structure interaction matrix, and, the subscript T refers to the transposed matrix.

This matrix system is not symmetric. Then, one can not directly apply conventional methods for extracting normal modes. To make it symmetric, one proposes applying the technique described in reference [3,4] which consists in multiplying equation (11) on the left by the matrix S as follows:

$$S = \begin{bmatrix} K_S{}^T M_S{}^{-1} & 0 \\ -A^T M_S{}^{-1} & I \end{bmatrix}. \tag{8}$$

One obtains the symmetric matrix system:

$$\left[\overline{K} - \omega^2 \overline{M}\right]\overline{X} = \overline{F}, \tag{9}$$

where:

$$\overline{X} = \begin{Bmatrix} U \\ P \end{Bmatrix}, \overline{K} = \begin{bmatrix} K_S{}^T M_S{}^{-1} K_S & -K_S{}^T M_S{}^{-1} A \\ -A^T M_S{}^{-1} K_S & K_F + A^T M_S{}^{-1} A \end{bmatrix}, \overline{M} = \begin{bmatrix} K_S{}^T & 0 \\ 0 & M_F \end{bmatrix}, \overline{F} = \begin{Bmatrix} K_S{}^T M_S{}^{-1} F \\ -A^T M_S{}^{-1} F + Q \end{Bmatrix}. \tag{10}$$

Considering a FE model with lumped masses, the inversion of the mass matrix of the structure that occurs in these expressions is straightforward, avoiding the use of a numerical inversion process which consumes computational resources.

As $\overline{M}$ and $\overline{K}$ matrices are symmetric, one can write the generalised eigenvalue problem:

$$\left[\mathrm{Re}\{\overline{K}\} - \lambda\, \mathrm{Re}\{\overline{M}\}\right]\overline{X} = 0. \tag{11}$$

From a modal extraction method, the Θ first eigenvalues λ_n and the associated mass-normalized eigenvectors ϕ_n are numerically computed such that:

$$\phi_n{}^T\, \mathrm{Re}\{\overline{M}\}\phi_n = 1,\ \ \phi_n{}^T\, \mathrm{Re}\{\overline{K}\}\phi_n = \lambda_n,\ \ n \in [1,2,..,\Theta]. \tag{12}$$

In a second step and in order to improve the convergence of modal series, one introduces residual mode shapes with the technique described in [12,13]. It consists in enriching the modal basis with "quasi-static" responses of the system for the different excitations, and then to orthogonalize the new basis. In our case, to calculate the PTF of subsystem 1, one considers the N excitations corresponding to the successive excitation of the N patches and the external excitation of the structure to calculate pressures blocked.

At a specific angular frequency ω_c, one calculates the residual shapes φ_i due to the $N+1$ excitations $\overline{F}_i$:

$$\left[\mathrm{Re}\{\overline{K}\} - \omega_c{}^2\, \mathrm{Re}\{\overline{M}\}\right]\varphi_i = \overline{F}_i\,. \tag{13}$$

From these residual shapes, a new reduction basis P is defined as:

$$P = \{\phi_1 \dots \phi_\theta | \varphi_1 \dots \varphi_{N+1}\} \tag{14}$$

This basis is then re-orthogonalized with respect to the mass matrix, $\bar{M}$ and the stiffness matrix, $\bar{K}$. λ'_α and χ_α , $\forall \alpha \in [1, \Theta + N + 1]$ are, respectively, the new eigenvalues and the new eigenvectors which are mass-normalized.

Then, to estimate the forced response $\bar{X}$ from Eq. (13) due to the excitation, F_i , an approximate solution can be found in the new basis $P' = \{\chi_1 \dots \chi_{\Theta+N+1}\}$,

$$\bar{X} = P'\Gamma \tag{15}$$

where Γ is the vector of the modal coordinates.

To this end, this expression is introduced in Eq. (13) and the resulting equation is projected in the P' basis.

By neglecting the off-diagonal terms of the imaginary part of modal matrices, and by introducing the modal damping factors, ζ_α, η_α and the generalized force, $F_{i\alpha}$ defined as follows:

$$\zeta_\alpha = \chi_\alpha{}^T \operatorname{Im}(\bar{M})\chi_\alpha, \quad \eta_\alpha = \chi_\alpha{}^T \operatorname{Im}(\bar{K})\chi_\alpha, \quad F_{i\alpha} = \bar{F}_i \chi_\alpha \tag{16}$$

one obtains the modal coordinates Γ_α:

$$\Gamma_\alpha = \frac{F_{i\alpha}}{-(1 + j\zeta_\alpha)\omega^2 + (1 + j\eta_\alpha)\lambda'_\alpha}, \forall \alpha \in [1, \Theta + N + 1] \tag{17}$$

The response of the structure-cavity system is then calculated from Eqs. (19,21) and the modal information $(\lambda'_\alpha, \chi_\alpha)$. DMAP procedure was written to perform the calculations of the coupled modes and the residual modes in the MSC/NASTRAN code.

It may be noted that this non-standard modal decomposition for fluid-structure system shows two damping factors for each mode, ζ_α, η_α. Their values depend on the damping factors associated to the structure and the cavity, and on the spatial distribution of the mode shapes.

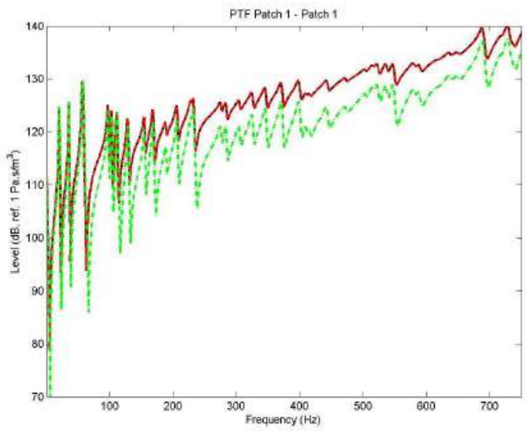

Figure 4. Comparison of three methods for estimating the input patch transfer function: dash-dotted line, modal superposition without residual shapes; dash line, modal superposition with residual shapes; solid line, direct FEM results.

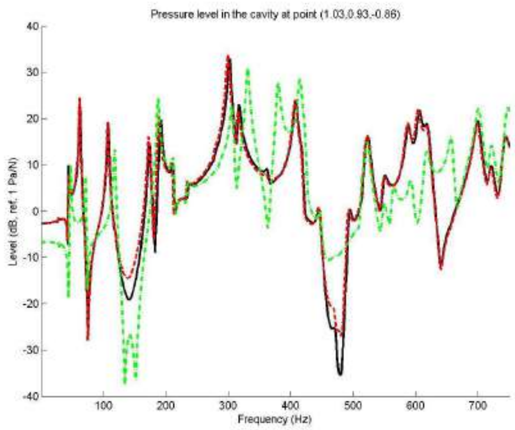

Figure 5. Comparison between three calculations of the pressure level in the cavity of the test case: dash-dotted line, PTF results using modal superposition without residual shapes; dash line, PTF results using modal superposition taking the residual shapes into account; solid line, direct FEM results.

For the test case described previously, one proposes on Fig. 4 to compare three calculations of the input patch transfer function of patch 1 of subsystem 1: a reference calculation obtained by a direct resolution of the finite element problem, a second calculation by the modal superposition method described in this section without considering the residual modes and finally, a third calculation using the modal superposition method taking into account the residual modes as described in this section. For these calculations, we consider the normal modes with a natural frequency below 1500 Hz (ie 100 modes) and the specific pulsation ω_c for calculating the residual shapes is set to 314 rad/s. It can be seen on Fig. 4 that the residual modes can significantly improve the convergence of modal expansions. A modification of the specific pulsation ω_c does not alter these results as far as this one does not correspond to a natural pulsation of the considered subsystem. One emphasizes that the use of residual modes does not increase significantly the calculation times.

These PTFs calculated from the modal method are then used in the PTF approach for estimating the global response of the test case. The acoustic pressure inside the cavity obtained from the PTF approach is compared with the reference results in Figs. 5. One can notice that the poor convergence of the PTFs calculated without the residual shape modes leads to significant errors in the PTF calculation of the acoustic pressure inside the cavity. On the other side, the use of residual modes yields results close to the reference result. A significant decrease in the computing times is obtained with the proposed PTF approach compared to a direct FE calculation.

5. CONCLUSIONS

One has shown that the PTF approach can be an efficient tool for modelling the heavy fluid - structure interaction. The optimal process has been obtained by substructuring the structure-cavity system outside the near-field zone of the structure and by using a non-standard modal decomposition for estimating the PTFs of the subsystem composed by the structure and the surrounding fluid. The approach can be used, for example, for estimating the sound transmission though bulkheads in the Sonar cavity of a submarine.

REFERENCES

[1] N. Roy, A. Girard. “Impact of residual modes in structural dynamics”. *Proc. of SSMMT*, Noordwijk, Hollande (2005).

[2] N. Roy, M. Lapi. “Efficient Computation of the Radiated Sound Power of Vibrating Structures using Modal Approach”. *Proc. of Acoustics’08*, Paris, France (2008).

[3] B. Tiliouine, A. Seghir. “Sur la symétrisation du problème de vibrations des systèmes couples fluide-structure”. *Proc. of AFPS*, Cachan, France (1999).

[4] B. Tiliouine, A. Seghir. “A numerical model for time domain analysis of dams including fluid-structure interaction”. *Proc. of CST98*, Edinburg, Écosse (1998).

[5] X. Wang, K.J. Bathe. “Displacemeent/Pressure Based Mixed Finite Element Formulations for Acoustic Fluid Structure Interaction Problems”. *Int. J. Numer. Meth. Engng*, 40, 2001–2017 (1997).

[6] J.F. Sigrist, S. Garreau. “Dynamic analysis of fluid-structure interaction problems with modal methods using pressure-based fluid finite elements“. *Finite Elmts in Anal. and Design*, 43, 287-300 (2007).

[7] J.F. Sigrist. “Symmetric and non-symmetric formulations for fluid-structure interaction problems: reference test cases for numerical developments in commercial finite element code”. *Proc. of Pressure Vessel and Piping*, Vancouver, Canada (2006).

[8] R. Ohayon. “Reduced models for fluid-structure interaction problems”. *Int. J. Numer. Meth. Engng*. 60, 139-152 (2004).

[9] Q.H. Tran, M. Ouisse, N. Bouhaddi. “Comparison of Robust CMS Methods for Damped Vibr-acoustic Problems”. *Proc. of ISMA.* Leuven, Belgique (2008).

[10] M Ouisse, L Maxit, C Cacciolati, and Guyader J-L. Patch Transfer Functions as a Tool to Couple Linear Acoustic Problems. *Journal Vibration-Acoustics*, 127, 458–466 (2005).

[11] N. Totaro, B. Andro, C. Peteul, J.L. Guyader. “Extension of the Patch Transfer Functions method to high frequency domain (sub-cavities decomposition)”. *Internoise 2007*, Istanbul, Turquie (2007).

[12] M. Aucejo, L. Maxit, N. Totaro, J.L. Guyader, “Introduction of residual modes concept in the Patch Transfer Functions method to model the structure-acoustic coupling in heavy fluid”. *Proc. of ICSV16*, Krakow, Pologne (2009).

[13] E. Balmès. ”Theoretical Course Notes: Methods for Vibration Design and Validation”. In: *Ecole Centrale Paris*, France (2008).